جامعة بنها- كلية العلوم

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نموذج اجابة

المادة: ميكانيكا الكم1

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أجابة الاسئلة

Answer of questions

Answer First Question:

1- where $\hat{A}e_1 = -e_1, \quad \hat{A}e_2 = -e_2, \quad \hat{A}e_3 = -e_3,$

$$\hat{B}e_1 = \sqrt{\frac{1}{2}}e_1 + \sqrt{\frac{1}{2}}e_2, \quad \hat{B}e_2 = \sqrt{\frac{1}{2}}e_1 - \sqrt{\frac{1}{2}}e_2, \quad \hat{B}e_3 = e_3$$

then

$$\left[\hat{A}, \hat{B}\right]e_1 = \quad \hat{A}\left(\sqrt{\frac{1}{2}}e_1 + \sqrt{\frac{1}{2}}e_2\right) - \hat{B}\left(\sqrt{\frac{1}{2}}e_1 + \sqrt{\frac{1}{2}}e_2\right).$$

$$-\left(\sqrt{\frac{1}{2}}e_1 + \sqrt{\frac{1}{2}}e_2\right) - \left(\sqrt{\frac{1}{2}}e_1 + \sqrt{\frac{1}{2}}e_2\right) - \sqrt{\frac{1}{2}}e_1 - \sqrt{\frac{1}{2}}e = 0$$

As the same

$$\left[\hat{A}, \hat{B}\right]e_2 = 0. \qquad \left[\hat{A}, \hat{B}\right]e_3 = 0.$$

2-To Determine the mean value of a mechanical quantity $\hat{L}_z^{\,2}$ described by the hermitian operator $\hat{L}_z^{\,2} = h^2 \frac{\partial^2}{\partial \varphi^2}$ in the state

$\Phi(\varphi) = A\sin^2\varphi, \quad 0 \le \varphi \le 2\pi$

Let $\qquad \|\Phi(\varphi)\|^2 = 1 = A^2 \int_0^{2\pi} \sin^4\varphi \quad d\varphi,$

$$\Rightarrow A^2 = \frac{4}{3\pi}, \Rightarrow A = \frac{2}{\sqrt{3\pi}}$$

Then $\qquad \Phi(\varphi) = \frac{2}{\sqrt{3\pi}}\sin^2\varphi, \quad 0 \le \varphi \le 2\pi$

$$\therefore \quad \left\langle L_z^2 \right\rangle_\Phi = \left(\Phi \middle| L_z^2 \Phi\right) = \left(\frac{4}{3\pi}\right)\left(-h^2\right)\int_0^{2\pi} \sin\varphi \left(\frac{\partial^2}{\partial\varphi^2}\sin^2\varphi\right) d\varphi$$

$$= \frac{4h^2}{3}$$

Answer Second Question:

1-The eigenvalues of $\hat{\pi}$

Let $\hat{\pi}\,\Psi_a = a\Psi_a, \qquad \Rightarrow \hat{\pi}^{\,2}\Psi_a = a^2\Psi_a = \Psi_a,$

Then $(a^2 - 1)\Psi_a = 0, \quad \Rightarrow a^2 - 1 = 0 \Rightarrow a = \pm 1$ since $\Psi_a \neq 0,$

the eigenvectors of $\hat{\pi}$

$$a = 1 \quad \Rightarrow \Psi_a = \Psi_+(x)$$

$$\hat{\pi}\,\Psi_a = +\Psi_+(x)$$

$$\hat{\pi}\Psi_+ = \Psi_+(-x) \Rightarrow \Psi_+(x) = \Psi_+(-x) \quad *$$

i.e. the eigenvector of corresponding $a = 1$ are all element satisfies (*) these element is said even parity

$$a = -1 \quad \Rightarrow \Psi_a = \Psi_-(x)$$

$$\hat{\pi}\,\Psi_a = -\Psi_-(x)$$

$$\hat{\pi}\Psi_+ = \Psi_+(-x) \Rightarrow \Psi_-(x) = -\Psi_-(-x) \quad **$$

i.e. the eigenvector of corresponding $a = -1$ are all element satisfies (**) these element is said odd parity

To show that the eigenvector of its form a complete set we make

$$\forall\Psi \in L_2 \quad \Rightarrow \Psi = \frac{1}{2}(\Psi(x) + \Psi(-x)) + \frac{1}{2}\left(\Psi(x) - \Psi(-x)\right)$$

Where the first term is even and the second term is odd then we can take

$$\Psi_+ = \frac{1}{2}(\Psi(x) + \Psi(-x))$$

$$\Psi_- = \frac{1}{2}\left(\Psi(x) - \Psi(-x)\right)$$

i.e. the eigenvector of $\hat{\pi}$ form a complete set

2- $\because \quad \{\Psi_n(x)\}$ are eigenvector of $\widehat{A}, \quad \widehat{B}$ then

Let $\widehat{A}\,\Psi_n = a_n\Psi_n, \qquad \widehat{B}\,\Psi_n = b_n\Psi_n,$

$$\widehat{A}\,\widehat{B}\,\Psi_n = b_n a_n \Psi_n, \qquad \widehat{B}\,\widehat{A}\,\Psi_n = a_n b_n \Psi_n,$$

Since $b_n\, a_n = a_n b_n$ then

$$\widehat{A}\,\widehat{B}\,\Psi_n = \widehat{B}\,\widehat{A}\,\Psi_n,$$

Where $\{\Psi_n(x)\}$ is a complete set

$$\forall\Psi \in H \Rightarrow \Psi = \sum_n \alpha_n \Psi_n \quad \text{then} \qquad \widehat{A}\,\widehat{B}\,\Psi = \widehat{B}\,\widehat{A}\,\Psi \Rightarrow \left[\widehat{A}, \widehat{B}\right] = 0$$

Answer Third Question:

To find the eigenfunction and corresponding eigenvalues we make

$\because \quad U(x)=\begin{cases}0 & -a\le x\le a\\ \infty & x\le -a,\, x\ge a\end{cases}$. What the allowed energy values

Then to find the solution of sch. Equation

$$\frac{d^2\Psi(x)}{dx^2}+\frac{2m}{h}[E-U(x)]\Psi(x)=0$$

We have two cases:

a-For the region inside the box

i.e. $-a\le x\le a$, sch. Equation is

$$\frac{d^2\Psi(x)}{dx^2}+k \quad \Psi(x)=0, \quad k=\sqrt{\frac{2mE}{h^2}} \qquad (1)$$

the general solution is

$$\Psi(x)=A\,e^{ikx}+B\,e^{-ikx} \qquad (2)$$

b-outside the box

the sch. Equation take the form

$$\frac{d^2\Psi(x)}{dx^2}+\infty \quad \Psi(x)=0, \qquad (3)$$

Where $\Psi(x)$ is finit then the only solution of equation is the trivaial solution

$$\Psi(x)=0, \qquad for \quad |x|\ge a \qquad (4)$$

Where $\Psi(x)$ is continuous thus

$$\Psi(-a)=0, \qquad \Psi(a)=0 \qquad (5)$$

then from equations 2 and 5, we have

$$A=-Be^{2ika}\Rightarrow \sin 2ka=1\Rightarrow k=\frac{n\pi}{2a}, \quad n=1,2,3,\ldots \quad (*)$$

Then from (*) we have

$$\Rightarrow K^2=\frac{2mE}{h^2}\Rightarrow E_n=\frac{(n\pi h)^2}{8ma^2}$$

also from (*) we have

$$\begin{aligned}\Psi(x)&=2A\cos kx, \quad n-odd\\ \Psi(x)&=2Ai\sin kx, \quad n-even\end{aligned} \qquad (6)$$

and from the normalization condition $\|\Psi(x)\|^2=1$ we have

$A=\frac{1}{\sqrt{2a}}$ i.e. the eigenstate is given by

$$\Psi(x)=\frac{1}{\sqrt{a}}\cos\frac{n\pi}{2a}x, \quad n-odd$$

$$\Psi(x)=\frac{1}{\sqrt{a}}\sin\frac{n\pi}{2a}x, \quad n-even$$

Answer the fourth Question:

1-To show the expection value $\langle \hat{p} \rangle_{\Phi_i}$, of the $\hat{p}$ momentum vanishes

Let $$\hat{H}\,\Phi_n(x)=E_n(x)\Phi_n(x), \qquad (\Phi_n,\Phi_m)=\delta_{mn}$$

$$\therefore \quad \langle P \rangle_{\Phi_n}=\left(\Phi_n\middle|\hat{P}\Phi_n\right),$$

Since $\left[\hat{x},\hat{P}\right]=ih$ then

$$\left[\hat{H},\hat{x}\right]=\frac{1}{2m}\left[\hat{P}^2,\hat{x}\right]=-\frac{ih}{m}\hat{P}$$

$$\Rightarrow \frac{im}{h}\left[\hat{H},\hat{x}\right]=\hat{P}$$

$$\therefore \quad \langle P \rangle_{\Phi_n}=\left(\Phi_n\middle|\hat{P}\Phi_n\right)=\left(\Phi_n,\frac{im}{h}\left[\hat{H},\hat{x}\right]\Phi_n\right)=\frac{im}{h}\left(\Phi_n,\middle|\hat{H}x-x\hat{H}\middle|\Phi_n\right)$$

$$=\frac{im}{h}\{(\Phi_n,Hx\Phi_n)-(\Phi_n,xH\Phi_n)\}=\frac{im}{h}\{E_n-E_n\}(\Phi_n,x\Phi_n)=0$$

2- $\hat{A}=k(\sin x)-i(\cos x)\frac{d}{dx}, \quad \hat{B}=k(\cos x)+i(\sin x)\frac{d}{dx}, \quad \hat{C}=-i\frac{d}{dx}$ i-

$\therefore \left[\hat{A},\hat{B}\right]=i\hat{C},$

$\left[\hat{B}, \quad \hat{C}\right]=-i\hat{A},$

$[\hat{C}, \quad \hat{A}]=-i\hat{B}$

انتهت الاجابة

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