جامعة بنـها - كلية العلوم – قسم الرياضيات

لطلاب المستوى الثانى

يوم الامتحان: السبت ١٦ / ١ / ٢٠١٦ م

المادة : رياضيات متقطعة (٢٢٥ ر)

الممتحن: د . / محمد السيد عبدالعال عبدالغنى

مدرس بقسم الرياضيات بكلية العلوم

اسئله + نموذج إجابه

ورقة كاملة

رياضيات متقطعة (٢٢٥ ر) لطلاب المستوى الثانى

Answer the following questions: (80 marks) أجب على الاسئله التاليه (الدرجة الكلية ٨٠ درجة)

Question 1. السؤال الأول (35 درجة) :-

1- Let $A = P\{1,2,3\}$, the power set of $\{1,2,3\}$ and $a\,R\,b$ *if and only if* $a \subseteq b$ be a relation on A. **Write down** its binary matrix. **Determine** which of the properties, reflexive, symmetric, transitive, the relation R is satisfied.

1- **State** the *converse*, *inverse* and *contrapositive* of the proposition: '***If it's not Sunday then the supermarket is open until midnight***'.

2- Let A, B, C are sets, **prove that:**

I. $(A-C)-(B-C)=(A-C)-B$

II. $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.

3- For any propositions *p*, *q*, *r*, **Prove that:** $(p\leftrightarrow q)\leftrightarrow r \equiv p\leftrightarrow(q\leftrightarrow r)$.

Question 2. السؤال الثانى (25 درجة)

1. A relation R on $Z^+\times Z^+$ is defined by $(m,n)R(p,q)$ if and only if $m+q=n+p$. Show that R is an equivalence relation and describe the equivalence class of (2,1).

2. **draw** diagram to represent the graph whose adjacency matrix is given below. **Write down** the degree of each vertex, and **state** the graph is (a) *simple* ; (b) *regular* ; (c) *Eulerian*.

$$A=\begin{pmatrix}1&2&0&2&1\\2&1&2&0&1\\0&2&1&2&1\\2&0&2&1&1\\1&1&1&1&1\end{pmatrix}$$

3. **Define** a Boolean algebra $(B,\oplus,*,\bar{\ },0,1)$ and for all $b_1,b_2\in B$, **prove that**:

$(\overline{b_1 * b_2}) = \overline{b_1}\oplus\overline{b_2}$.

انظر خلف الورقة

جـــامعة بنـــــها
كلـية الــــعلوم
قسـم الرياضيات

نموذج اجابة امتحان
رياضيات متقطعة (٢٢٥ ر)

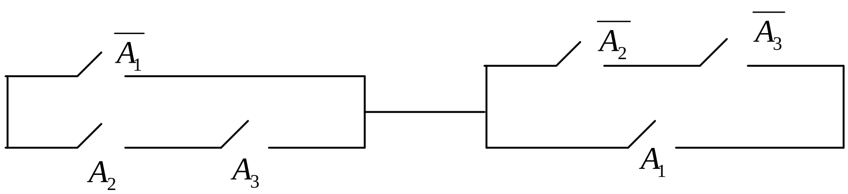
Question 3.

السؤال الثالث (20 درجة):

1. **Define** *Hamiltonian cycle*, *r- regular* graph, a *connected graph G*, and **how** can you determine from its adjacency matrix, whether or not *G* is ***Eulerian*.**

2. **Show** the following function is a **bijection** and find its **inverse**:
   $f : R \to R,\ f(x) = (5x-3)^3 \ \ \forall x \in R.$

3. **Define** a switching function for the following system of switches:



4. **Design** a logic network for the following so that the output is described by the following Boolean expression: $(x_1 \oplus x_2)(\overline{x_1} \oplus \overline{x_2})$.

انتهت أسئلة

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مع أطيب تمنياتى بالتوفيق والنجاح
د. محمد السيد عبدالعال

Good Luck !

نموذج اجابه لأمتحان رياضيات متقطعة (٢٢٥ ر) لطلاب المستوى الثانى

(الدرجة الكلية ٨٠ درجة)

اجابة السؤال الأول (٣٥ درجة) :-

2- Let $A = P\{1,2,3\}$, the power set of $\{1,2,3\}$ and $a\ R\,b\ if\ and\ only\ if\ \ a \subseteq b$ be a relation on A. Write down its binary matrix. Determine which of the properties, reflexive, symmetric, transitive, the relation R is satisfied.

الحـــــــــــــل

$$A = \begin{pmatrix} & \phi & \{1\} & \{2\} & \{3\} & \{1,2\} & \{1,3\} & \{2,3\} & \{1,2,3\} \\ \phi & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \{1\} & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ \{2\} & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ \{3\} & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ \{1,2\} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ \{1,3\} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ \{2,3\} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \{1,2,3\} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

***R* is reflexive , anti-symmetric , and transitive.**

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3- **State** the *converse*, *inverse* and *contrapositive* of the proposition: '***If it's not Sunday then the supermarket is open until midnight***'.

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We define: p: ***it's not Sunday***
q: ***the supermarket is open until midnight***
so that:
$p \to q$: ***If it's not Sunday then the supermarket is open until midnight***'.

Converse
: $q \to p$: If ***the supermarket is open until midnight***' then ***it's not Sunday***.

Inverse
: $\sim p \to \sim q$: ***If it's Sunday*** then ***the supermarket is not open until midnight***'.

Contrapositive

: $\sim q \to \sim p$: If ***the supermarket is not open until midnight*** then ***it's Sunday.***

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4- Let A , B , C are sets, **prove that:**

I. $(A-C)-(B-C)=(A-C)-B$

II. $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.

الحـــــــــــل

$$(A-C)-(B-C)=(A\cap\overline{C})-(B\cap\overline{C})=(A\cap\overline{C})\cap\overline{(B\cap\overline{C})}$$

$$=(A\cap\overline{C})\cap(\overline{B}\cup\overline{\overline{C}})=(A\cap\overline{C})\cap(\overline{B}\cup C)$$

$$=[(A\cap\overline{C})\cap\overline{B}]\cup[(A\cap\overline{C})\cap C]$$

$$=[(A-C)-B]\cup\phi=(A-C)-B$$

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II. $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

$$A\cup(B\cap C)\Leftrightarrow\{x\in A\vee x\in(B\cap C)\}$$

$$\Leftrightarrow\{x\in A\vee[x\in B\wedge x\in C)\}$$

$$\Leftrightarrow\{[x\in A\vee x\in B]\wedge[x\in A\vee x\in C]\}$$

$$\Leftrightarrow\{[x\in(A\cup B)]\wedge[x\in(A\cup C]\}$$

$$\Leftrightarrow\{x\in[(A\cup B)\cap(A\cup C)\}$$

Then $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

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4. For any propositions p, q, r, **Prove that:** $(p\leftrightarrow q)\leftrightarrow r\equiv p\leftrightarrow(q\leftrightarrow r)$..

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p	**q**	**r**	$p\leftrightarrow q$	$(p\leftrightarrow q)\leftrightarrow r$	$q\leftrightarrow r$	$p\leftrightarrow(q\leftrightarrow r)$
1	**1**	**1**	**1**	**1**	**1**	**1**
1	**1**	**0**	**1**	**0**	**0**	**0**
1	**0**	**1**	**0**	**0**	**0**	**0**
1	**0**	**0**	**0**	**1**	**1**	**1**

0	1	1	0	0	1	0
0	1	0	0	1	0	1
0	0	1	1	1	0	1
0	0	0	1	0	1	0

Thus $(p \leftrightarrow q) \leftrightarrow r \equiv p \leftrightarrow (q \leftrightarrow r)$

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أجابة السؤال الثانى (25 درجة) :-

1. *A relation R on* $Z^+ \times Z^+$ *is defined by* $(m,n)R(p,q)$ *if and only if* $m+q=n+p$. *Show that R is an equivalence relation and describe the equivalence class of (2,1).*

الحــــــــــــل

For all positive integers *a* and *b*, *a* + *b* = *b* + *a*, so *(a, b) R (a, b)* for every *(a, b)* ∈ *A*, Therefore *R* is reflexive.
R is symmetric since if (*a*, *b*) *R* (*c*, *d*) then *a* + *d* = *b* + *c* which implies that *c* + *b* = *d* + *a*, so (*c*, *d*) *R* (*a*, *b*).
To show that *R* is transitive, suppose *(a, b) R (c, d)* and (*c*, *d*) *R* (*e*, *f*). This means that *a* + *d* = *b* + *c* and *c* + *f* = *d* + *e*.

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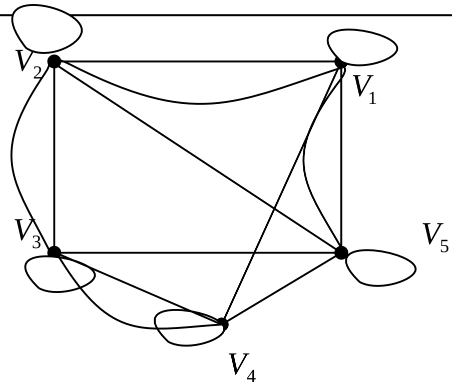
2. **draw** diagram to represent the graph whose adjacency matrix is given below. **Write down** the degree of each vertex, and **state** the graph is (a) *simple* ; (b) *regular* ; (c) *Eulerian*.

$$A=\begin{pmatrix} 1 & 2 & 0 & 1 & 1 \\ 2 & 1 & 2 & 0 & 1 \\ 0 & 2 & 1 & 2 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

الحــــــــــــل



the graph is not *simple* ; not *regular* ; not *Eulerian*.

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3. **Define** a Boolean algebra $(B, \oplus, *, \bar{\ }, 0, 1)$ and for all $b_1, b_2 \in B$, **prove that:** $\overline{(b_1 * b_2)} = \overline{b_1} \oplus \overline{b_2}$.

الحـــــــــــــل

Boolean algebra consists of a set *B* together with three operations defined on that set. These are:
(a) a binary operation denoted by ⊕ referred to as the **sum** ;
(b) a binary operation denoted by ∗ referred to as the **product** ;
(c) an operation which acts on a single element of *B*, denoted by -, where, for any element *b* ∈ *B*, the element b^{-} ∈ *B* is called the

complement of b^{-} (An operation which acts on a single member of a set *S* and which results in a member of *S* is called a **unary operation**.)

The following axioms apply to the set *B* together with the operations ⊕, ∗ and - .
B1. Distinct identity elements belonging to *B* exist for each of the binary operations ⊕ and ∗ and we denote these by **0** and **1** respectively. Thus we have

$$b \oplus \mathbf{0} = \mathbf{0} \oplus b = b$$
$$b * \mathbf{1} = \mathbf{1} * b = b \quad \text{for all } b \in B.$$

for all *a*, *b*, *c* ∈ *B*.

$$(a * b) * c = a * (b * c)$$
$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

B2. The operations ⊕ and ∗ are associative, that is
B3. The operations ⊕ and ∗ are commutative, that is

$$a \oplus b = b \oplus a$$

$$a * b = b * a \quad \text{for all } a, b \in B.$$

B4. The operation ⊕ is distributive over ∗ and the operation ∗ is distributive over ⊕, that is

$$a \oplus (b * c) = (a \oplus b) * (a \oplus c)$$

$$a * (b \oplus c) = (a * b) \oplus (a * c) \text{ for all } a, b, c \in B.$$

B5. For all $b \in B$, $b \oplus .b = 1$ and $b * .b = 0$.

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$(b_1 \oplus b_2) \oplus (\overline{b_1} * \overline{b_2}) = [(b_1 \oplus b_2) \oplus \overline{b_1}] * [(b_1 \oplus b_2) \oplus \overline{b_2}]$ (axiom B4)

$= [\overline{b_1} \oplus (b_1 \oplus b_2)] * [(b_1 \oplus b_2) \oplus \overline{b_2}]$ (axiom B3)

$= [(\overline{b_1} \oplus b_1) \oplus b_2] * [b_1 \oplus (b_2 \oplus \overline{b_2})]$ (axiom B2)

$= (1 \oplus b_2) * (b_1 \oplus 1)$ (axiom B5)

$= 1 * 1$ (theorem 9.4)

$= 1$ (axiom B1).

We have proved that $(b_1 \oplus b_2) \oplus \overline{b_1} * \overline{b_2}) = 1$ so that $\overline{b_1} * \overline{b_2}$ is the complement of $b_1 \oplus b_2$, i.e. $(b_1 \oplus b_2) = \overline{b_1} * \overline{b_2}$.

That $(b_1 * b_2) = \overline{b_1} \oplus \overline{b_2}$ follows from the duality principle.

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أجابة السؤال الثالث (20 درجة) :-

1. **Define** *Hamiltonian cycle, r- regular* graph, a *connected graph G*, and **how** can you determine from its adjacency matrix, whether or not *G* is ***Eulerian.***

الحـــــــــــــــل

A graph is connected if, given any pair of distinct vertices, there exists a path connecting them.

A Hamiltonian cycle in a graph is a cycle which passes once through every vertex. A graph is Hamiltonian if it has a Hamiltonian cycle.

A graph in which every vertex has the same degree r is called regular (with degree r) or simply r-regular.

A graph is Eulerian if the sum of all entry in any row or in any column of its adjacency matrix is even.

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2. **Show** the following function is a **bijection** and find its **inverse**:

$$f : R \to R,\ f(x) = (5x-3)^3 \ \ \forall x \in R.$$

الحـــــــــــــــل

To show that *f* is an injection we prove that, for all real numbers *x* and *y*, *f (x) = f (y)* implies *x = y*. Now *f (x) = f (y)*

$(5x-3)^3 = .(5y-3)^3 \Rightarrow x = y$ so f is injective.

To show that f is a surjection, let y be any element of the codomain f . We need to find x ∈ R such that f (x) = y. Let $x = \frac{\sqrt[3]{y}+3}{5}$. Then x ∈ R and

$f(x) = [5\frac{\sqrt[3]{y}+3}{5} - 3]^3 = y$ so f is surjective.

To find f^{-1} we simply use its definition: if y = f (x) then x = f^{-1}(y).

Now y = f (x)

$\Rightarrow y = (5x-3)^3 \Rightarrow x = \frac{\sqrt[3]{y}+3}{5}$

$x = f^{-1}(y) = \frac{\sqrt[3]{y}+3}{5}$. Therefore the inverse function is $f^{-1} : R \to R$, $f^{-1}(y) = \frac{\sqrt[3]{y}+3}{5}$.

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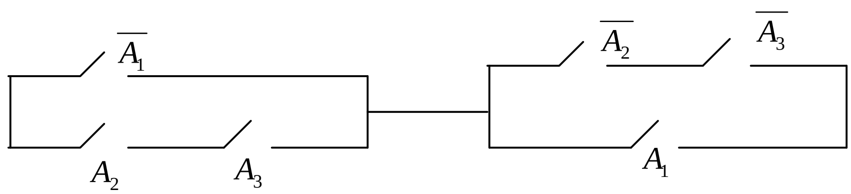
3. **Define** a switching function for the following system of switches:



$$f(x_1, x_2, x_3) = (\overline{x_1} \oplus x_2 x_3)(\overline{x_2}\,\overline{x_3} \oplus x_1).$$

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4. **Design** a logic network for the following so that the output is described by the following Boolean expression: $(x_1 \oplus x_2)(\overline{x_1} \oplus \overline{x_2})$.