جـــامعه بنـــــها
كلـيه الــــــعلوم
قسـم الرياضيات

الزمن: ساعتان
الترم الأول
٢٠١٥/ ٢٠١٦

إجابة إمتحان قسم علوم الحاسب للفرقة الرابعة (تطبيقات الحاسب فى الرياضيات)

إجابة السؤال الأول:

أ- أستنتج طريقة (Trapezoidal rule):

if we want to approximate the integral

$$I = \int_a^b f(x)dx$$

to find the value of the above integral, we write our function under polynomial form:

$$f(x) \approx f_n(x)$$

where

$$f_n(x) = a_0 + a_1 x + \dots + a_{n-1}x^{n-1} + a_n x^n$$

where $f_n(x)$ is an n^{th} order polynomial. **Trapezoidal rule assumes** $n = 1$, that is, the area under the linear polynomial (straight line),

$$\int_a^b f(x)dx \approx \int_a^b f_1(x)dx$$

$$= \int_a^b (a_0 + a_1 x)dx$$

$$.= a_0(b-a) + a_1\left(\frac{b^2 - a^2}{2}\right)$$

But what is a_0 and a_1? Now if we choose, $(a, f(a))$ and $(b, f(b))$ as the two points to approximate $f(x)$ by a straight line from a to b,

$$f(a) = f_1(a) = a_0 + a_1 a$$
$$f(b) = f_1(b) = a_0 + a_1 b$$

Solving the above two equations for a and b,

$$a_1 = \frac{f(b) - f(a)}{b-a}$$
$$a_0 = \frac{f(a)b - f(b)a}{b-a}$$

Hence we get,

$$\int_a^b f(x)dx = \frac{f(a)b - f(b)a}{b-a}(b-a) + \frac{f(b)-f(a)}{b-a}\frac{b^2-a^2}{2}$$

$$\boxed{\int_a^b f(x)\ dx = (b-a)\left[\frac{f(a)+f(b)}{2}\right]}$$

ب-اكتب برنامج يحسب التكامل $\int_1^2 xdx$ باستخدام (**Trapezoidal rule**):

```
function y = f(x)
y=x;

function integral = cmptrap(a,b,n,f)
h = (b-a)/n;
x = [a+h:h:b-h];
integral =
h/2*(2*sum(feval(f,x))+feval(f,a)+feval(f,b));
%Example: cmptrap(1,2,10,'f')
```

إجابة السؤال الثاني:

أ- استنتج طريقة (central, forward and backward finite difference) :

Forward Difference Approximation of the First Derivative

From differential calculus, we know

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

For a finite Δx,

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

The above is the forward divided difference approximation of the first derivative. It is called forward because you are taking a point ahead of x. To find the value of $f'(x)$ at $x = x_i$, we may choose another point Δx ahead as $x = x_{i+1}$. This gives

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_i)}{\Delta x} = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

where $\Delta x = x_{i+1} - x_i$

Backward Difference Approximation of the First Derivative

We know

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

For a finite Δx,

$$f'(x) \approx \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

If Δx is chosen as a negative number,

$$f'(x) \approx \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{f(x) - f(x - \Delta x)}{\Delta x}$$

This is a backward difference approximation as you are taking a point backward from x. To find the value of $f'(x)$ at $x = x_i$, we may choose another point Δx behind as $x = x_{i-1}$. This gives

$$f'(x_i) \approx \frac{f(x_i) - f(x_{i-1})}{\Delta x} = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

where

$$\Delta x = x_i - x_{i-1}$$

Central Difference Approximation of the First Derivative:

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x}$$

ب- اكتب برنامج يحسب تفاضل الدالة $f(x) = x\cos(x)$ باستخدام الطرق

:(central, forward and backward finite difference)

```
function f = f_ex( x );
f  = cos(x )-x*sin( x )  ;

function f = f_center( x , h  );
f1 =   cos(x+h)-(x+h)*sin(x+h)  ;
```

```
f2 =  cos(x-h)-(x-h)*sin(x-h) ;
f = ( f1-f2) / (2*h) ;

function f = f_forward( x , h );
f1 =   cos(x+h)-(x+h)*sin(x+h) ;
f2 =  cos(x)-(x)*sin(x) ;
f = ( f1-f2) / (h) ;

function f = f_backward( x , h );
f1 =   cos(x)-(x)*sin(x) ;
f2 =  cos(x-h)-(x-h)*sin(x-h) ;
f = ( f1-f2) / (h) ;
```

إجابة السؤال الثالث:

A- Write a program to find the approximate solution of the following initial value problem:

$$y'(x) = xy^2 + y, x \in [0, 0.5],$$

$$y(0) = 1$$

by using (Rung-Kutta $2^{nd}/3^{rd}$) method

```
function yprime = fode(x,y);
yprime = x*y^2 + y;
>>xspan = [0,.5];
>>y0 = 1;
>>[x,y]=ode23('fode',xspan,y0);
```

B- Write a program to find the exact solution of the above equation

```
>>y = dsolve('Dy = y*y*x+y','x')
```

C- Write a program to find the exact integral for the following indefinite integrations:

$$\int \frac{\tan^{-1}(x)}{1+x^2} dx, \qquad \int x e^{x^2} dx$$

```
syms x
I1=int(atan(x)./(1+x^2))
I2=int(x*exp(x*x))
```

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السؤال الرابع:

Write a program to find the approximate solution of the linear-advection equation
(dU/dt + v dU/dx = 0) on the interval [p=0,q=100] and v=0.7 and time step dt=0.3
by using the forward finite difference:

```
function linearadvection
clear all; clc; clf
p=0;
q=100;
v=0.7;
N=101;
dx=(q-p)/(N-1);
x = p: dx : q;
u0=zeros(1,N);
for i=1:N
    u0(i) = finitial(x(i));
end
dt=0.3;
ntimesteps=10;
r =v*dt/dx;
u=zeros(1,N);

for n=1:ntimesteps
    t=n*dt;
    u0=[u0  u0(N)];
    for i=1:N
        u(i)=u0(i)-r*(u0(i+1)-u0(i));
    end
     plot(x,u(1:N),'r+')
     xlabel('x')
     ylabel('U')
     title('numerical solution to dU/dt + v dU/dx = 
0')
     pause(0.3)
     u0=u(1:N);
     u=[];
end
```

```
function y = finitial(x)
y=0.0;
if and(x>= 20,x<=70)
   y = exp(-0.01*(x-45)^2);
end
```

مع أطيب التمنيات
د/هبة السيد فتحى